The Relativistic Electromagnetic Force

Ram Rachum

Received ______________________; accepted ______________________

Abstract

This article gives the equations of the electromagnetic force on a body in a Special Relativity world. This is derived from the Liénard-Wiechert Potentials.

We have two bodies, and we want to know what is the electromagnetic force that one of them exerts on the other. Let's call the receiver of the force Obj, and the maker of the force Src.

Now let's define our notation. Src's position, velocity and acceleration functions will be called $\vec{s}\,(t)$, $\vec{v}\,(t)$ and $\vec{a}\,(t)$, respectively. Its charge will be called q. The current time for which we want to measure the electromagnetic force will be called t. The position of Obj at time t will be called $\vec{r}$.

A few more definitions. Define T, the retarded time:

$$T = t - \frac{|\vec{r} - \vec{s}\,(T)\,|}{c} \tag{1}$$

You might have noticed that T appears on both sides of the preceding equation. This makes things complicated sometimes, but I assure you that T is perfectly well-defined as the only number which satisfies that equation.

Intuitively, the retarded time T is the time point in which the object Src might have sent a light ray that have hit Obj at precisely the moment t.

Define $\vec{R}$:

$$\vec{R} = \vec{r} - \vec{s}(T) \tag{2}$$

We will use $\hat{R}$ to denote $\frac{\vec{R}}{|\vec{R}|}$.

We will use the relativistic gamma function, notated like this:

$$\gamma_v = \left(1 - \frac{v^2}{c^2}\right)^{-0.5} \tag{3}$$

Now, the way we're going to calculate the electromagnetic force on Obj, is that we're going to calculate the electric and magnetic fields in the point where Obj is located. Given the fields, it's possible to calculate the force using the classic formula $\vec{\mathcal{F}} = \mathcal{Q}\left(\vec{E} + \vec{\mathcal{V}} \times \vec{B}\right)$, where $\mathcal{Q}$ is Obj's charge, $\vec{\mathcal{V}}$ is its velocity and $\vec{\mathcal{F}}$ is the force acting on it.

Without further ado, here are the equations:

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \cdot \frac{\left(\hat{R} - \frac{\vec{v}(T)}{c}\right)\gamma_{\vec{v}(T)}^{-2} + \frac{\vec{R}}{c} \times \left(\left(\hat{R} - \frac{\vec{v}(T)}{c}\right) \times \frac{\vec{a}(T)}{c}\right)}{\left(1 - \hat{R} \cdot \frac{\vec{v}(T)}{c}\right)^3 \left|\vec{R}\right|^2} \tag{4}$$

$$\vec{B} = \frac{\hat{R} \times \vec{E}}{c} \tag{5}$$